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ON INTERVAL VALUED INTUITIONISTIC (α,β)-FUZZY HV-SUBMODULES

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ABSTRACT
Atanassov introduced the notion of intuitionistic fuzzy sets as a generalization of the notion of fuzzy sets. In this paper we introduce the concept of an interval valued intuitionistic (α, β)-fuzzy H_v-submodule of an H_v-module by using the notion of "belongingness $(\in)$" and "quasi-coincidence (q)" of fuzzy points with fuzzy sets, where $\alpha \in \{\in, q\}$, $\beta \in \{\in, q, \in\vee q, \in\wedge q\}$ and, then we give the basic properties of these notions.

KEYWORDS: Hyperstructure, Hv-module, Fuzzy set, Intuitionistic fuzzy set, Interval valued intuitionistic (α, β)-fuzzy Hv-submodule.

Mathematics Subject Classification: 20N20.

INTRODUCTION
The concept of hyperstructure was introduced in 1934 by Marty [4]. Hyperstructures have many applications to several branches of pure and applied sciences. Vougiouklis [13] introduced the notion of H_v-structures, and Davvaz [1] surveyed the theory of H_v-structures. After the introduction of fuzzy sets by Zadeh [8], there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov [5] is one among them. For more details on intuitionistic fuzzy sets, we refer the reader to [6, 7].

The idea of quasi-coincidence of a fuzzy point with a fuzzy set, which is mentioned in [10], played a vital role to generate some different types of fuzzy subgroups. Bhakat and Das [11, 12] gave the concepts of (α, β)-fuzzy subgroups by using the notion of "belongingness ($\in$)" and "quasi-coincidence (q)" between a fuzzy point and a fuzzy subgroup, where α, β are any two of $\{\in, q, \in\vee q, \in\wedge q\}$ with $\alpha \neq \in\wedge q$, and introduced the concept of an $(\in, \in\vee q)$-fuzzy subgroup. In [15] Yuan, Li et al. redefined (α, β)-intuitionistic fuzzy subgroups. M. Asghari-Larimi [9] gave intuitionistic (α, β)-fuzzy H_v-submodules. Basing on [9], in this paper, we introduce the concept of an interval valued intuitionistic (α, β)-fuzzy H_v-submodule of an H_v-module and describe the characteristic properties.

The paper is organized as follows: in section 2 some fundamental definitions on H_v-structures and fuzzy sets are explored, in section 3 we introduce interval valued intuitionistic (α, β)-fuzzy H_v-submodules and establish some useful results.

BASIC DEFINITIONS
We first give some basic definitions for proving the further results.

Definition 2.1 [3] Let X be a non-empty set. A mapping $\mu : X \to [0,1]$ is called a fuzzy set in X. The complement of μ, denoted by μ^c, is the fuzzy set in X given by $\mu^c(x) = 1 - \mu(x) \quad \forall x \in X$.

Definition 2.2 [3] An intuitionistic fuzzy set A in a non-empty set X is an object having the form $A=\{(x,\mu_A(x),\lambda_A(x)):x\in X\}$, where the functions $\mu_A:X\to[0,1]$ and $\lambda_A:X\to[0,1]$ denote the degree of membership and degree of non membership of each element $x\in X$ to the set A respectively and $0\le\mu_A(x)+\lambda_A(x)\le 1$ for all $x\in X$. We shall use the symbol $A=\{\mu_A,\lambda_A\}$ for the intuitionistic fuzzy set $A=\{(x,\mu_A(x),\lambda_A(x)):x\in X\}$.

Definition 2.3 [3] Let $A=\{\mu_A,\lambda_A\}$ and $B=\{\mu_B,\lambda_B\}$ be intuitionistic fuzzy sets in X. Then

$(1)\ A\subseteq B\Leftrightarrow \mu_A(x)\le\mu_B(x)\ and\ \lambda_A(x)\le\lambda_B(x)$,

$(2)\ A^c=\{(x,\lambda_A(x),\mu_A(x)):x\in X\}$,

$(3)\ A\cap B=\{(x,\min\{\mu_A(x),\mu_B(x)\},\max\{\lambda_A(x),\lambda_B(x)\}):x\in X\}$,

$(4)\ A\cup B=\{(x,\max\{\mu_A(x),\mu_B(x)\},\min\{\lambda_A(x),\lambda_B(x)\}):x\in X\}$,

$(5)\ \square A=\{(x,\mu_A(x),\mu_A^c(x)):x\in X\}$,

$(6)\ \Diamond A=\{(x,\lambda_A^c(x),\lambda_A(x)):x\in X\}$.

Definition 2.4 [14] Let G be a non–empty set and $*:G\times G\to\wp^*(G)$ be a hyperoperation, where $\wp^*(G)$ is the set of all the non-empty subsets of G. Where $A*B=\bigcup_{a\in A,\,b\in B} a*b,\ \ \forall A,B\subseteq G$.

The $*$ is called weak commutative if $x*y\cap y*x\neq\phi,\quad \forall x,y\in G$.

The $*$ is called weak associative if $(x*y)*z\cap x*(y*z)\neq\phi,\quad \forall x,y,z\in G$.

A hyperstructure $(G,*)$ is called an H_v–group if

(i) $*$ is weak associative.

(ii) $a*G=G*a=G,\quad \forall a\in G$ (Reproduction axiom).

Definition 2.5 [14] An H_v–ring is a system $(R,+,\cdot)$ with two hyperoperations satisfying the ring-like axioms:

(i) $(R,+,\cdot)$ is an H_v-group, that is,

$((x+y)+z)\cap(x+(y+z))\neq\phi\quad\forall x,y\in R,$

$a+R=R+a=R\quad\forall a\in R;$

(ii) $(R,\cdot)$ is an H_v-semigroup;

(iii) $(\cdot)$ is weak distributive with respect to $(+)$, that is, for all $x,y,z\in R$,

$(x\cdot(y+z))\cap(x\cdot y+x\cdot z)\neq\phi,$

$((x+y)\cdot z)\cap(x\cdot z+y\cdot z)\neq\phi.$

Definition 2.6 [2] Let R be an H_v-ring. A nonempty subset I of R is called a left (resp., right) H_v-ideal if the following axioms hold:

(i) $(I,+)$ is an H_v-subgroup of $(R,+)$,

(ii) $R\cdot I\subseteq I$ (resp., $I\cdot R\subseteq I$).

Definition 2.7 [2] Let $(R,+,\cdot)$ be an H_v–ring and μ a fuzzy subset of R. Then μ is said to be a left (resp., right) fuzzy H_v-ideal of R if the following axioms hold:

$(1)\min\{\mu(x),\mu(y)\}\le\inf\{\mu(z):z\in x+y\}\forall x,y\in R,$

(2) For all $x, a \in R$ there exists $y \in R$ such that $x \in a + y$ and $\min\{\mu(a), \mu(x)\} \leq \mu(y)$,
(3) For all $x, a \in R$ there exists $z \in R$ such that $x \in z + a$ and $\min\{\mu(a), \mu(x)\} \leq \mu(z)$,
(4) $\mu(y) \leq \inf\{\mu(z) : z \in x \cdot y\}$ respectively $\mu(x) \leq \inf\{\mu(z) : z \in x \cdot y\} \quad \forall x, y \in R.$

Definition 2.8 [2] An intuitionistic fuzzy set $A = \{\mu_A, \lambda_A\}$ in R is called a left (resp., right) intuitionistic fuzzy H_v-ideal of R if following axioms hold:
(1) $\min\{\mu_A(x), \mu_A(y)\} \leq \inf\{\mu_A(z) : z \in x + y\} \forall x, y \in R,$
(2) For all $x, a \in R$ there exists $y, z \in R$ such that $x \in (a + y) \cap (z + a)$ and $\min\{\mu_A(a), \mu_A(x)\} \leq \min\{\mu_A(y), \mu_A(z)\},$
(3) $\mu_A(y) \leq \inf\{\mu_A(z) : z \in x \cdot y\}$ respectively $\mu_A(x) \leq \inf\{\mu_A(z) : z \in x \cdot y\} \quad \forall x, y \in R,$
(4) $\sup\{\lambda_A(z) : z \in x + y\} \leq \max\{\lambda_A(x), \mu_A(y)\} \forall x, y \in R,$
(5) For all $x, a \in R$ there exists $y, z \in R$ such that $x \in (a + y) \cap (z + a)$ and $\max\{\lambda_A(y), \lambda_A(z)\} \leq \max\{\lambda_A(a), \lambda_A(x)\},$
(6) $\sup\{\lambda_A(z) : z \in x \cdot y\} \leq \lambda_A(y)$ respectively $\sup\{\lambda_A(z) : z \in x \cdot y\} \leq \lambda_A(x) \quad \forall x, y \in R.$

Definition 2.12 [17] A nonempty set M is called an H_v-module over an H_v -ring R if (M, +) is a weak commutative H_v -group and there exists a map
$.: R \times M \to \wp^*(M), (r, x) \to r.x$ Such that for all $a, b \in R$ and $x, y \in M$, we have
$(a.(x + y)) \cap (a.x + a.y) \neq \phi,$
$((x + y).a) \cap (x.a + y.a) \neq \phi,$
$(a.(b.x)) \cap ((a.b).x) \neq \phi.$

Note that by using fuzzy sets, we can consider the structure of H_v -module on any ordinary module which is a generalization of a module.

Definition 2.13 [12] A fuzzy set μ in M is called a fuzzy H_v-submodule of M if
(1) $\min\{\mu(x), \mu(y)\} \leq \inf\{\mu(z) : z \in x + y\} \forall x, y \in M,$
(2) For all $x, a \in M$ there exists $y, z \in M$ such that $x \in (a + y) \cap (z + a)$ and $\min\{\mu(a), \mu(x)\} \leq \inf\{\mu(y), \mu(z)\},$
(3) $\mu(y) \leq \inf\{\mu(z) : z \in x \cdot y\}$ for all $y \in M$ and $x \in R.$

Definition 2.14 [16] An intuitionistic fuzzy set $A = \{\mu_A, \lambda_A\}$ in an H_v –module M over an H_v –ring R is said to be an intuitionistic fuzzy H_v-submodule of M if the following axioms hold:
(1) $\min\{\mu_A(x), \mu_A(y)\} \leq \inf\{\mu_A(z) : z \in x + y\}$ and $\max\{\lambda_A(x), \lambda_A(y)\} \geq \sup\{\lambda_A(z) : z \in x + y\}$ for all $x, y \in M,$
(2) For all $x, a \in M$ there exists $y \in M$ such that $x \in a + y$ and $\min\{\mu_A(a), \mu_A(x)\} \leq \mu_A(y)$ and $\max\{\lambda_A(a), \lambda_A(x)\} \geq \lambda_A(y),$
(3) For all $x, a \in M$ there exists $z \in M$ such that $x \in z + a$ and $\min\{\mu_A(a), \mu_A(x)\} \leq \mu_A(z)$ and $\max\{\lambda_A(a), \lambda_A(x)\} \geq \lambda_A(z),$

$(4)\mu_A(x) \le \inf\{\mu_A(z): z \in r \cdot x\}$ and $\lambda_A(x) \ge \sup\{\lambda_A(z): z \in r \cdot x\}$ for all $x \in M$ and $r \in R.$

Definition 2.9 [12]Let μ be a fuzzy subset of R. If there exist a $t \in (0,1]$ and an $x \in R$ such that

$$\mu(y) = \begin{cases} t & if \ y = x \\ 0 & if \ y \ne x \end{cases}$$

Then μ is called a fuzzy point with support x and value t and is denoted by x_t.

Definition 2.10 [12] Let μ be a fuzzy subset of R and x_t be a fuzzy point. (1) If $\mu(x) \ge t$, then we say x_t belongs to μ, and write $x_t \in \mu$.

(2) If $\mu(x) + t > 1$, then we say x_t is quasi-coincident with μ, and write $x_t q\mu$.

$(3) x_t \in \vee q\mu \Leftrightarrow x_t \in \mu \ or \ x_t q\mu..$

$(4) x_t \in \wedge q\mu \Leftrightarrow x_t \in \mu \ and \ x_t q\mu.$

In what follows, unless otherwise specified, α and β will denote any one of $\in, q, \in\vee q$ or $\in\wedge q$ with $\alpha \ne \in\wedge q$, which was introduced by Bhakat and Das [9].

By an interval number $\tilde{a}$ we mean an interval $\left[a^-, a^+\right]$ where $0 \le a^- \le a^+ \le 1.$ The set of all interval numbers is denoted by $D[0,1]$. We also identify the interval $[a,a]$ by the number $a \in [0,1].$

For the interval numbers $\tilde{a}_i = \left[a_i^-, a_i^+\right] \in D[0,1], i \in I,$ we define

$$\max\left\{\tilde{a}_i, \tilde{b}_i\right\} = \left[\max\left(a_i^-, b_i^-\right), \max\left(a_i^+, b_i^+\right)\right],$$

$$\min\left\{\tilde{a}_i, \tilde{b}_i\right\} = \left[\min\left(a_i^-, b_i^-\right), \min\left(a_i^+, b_i^+\right)\right],$$

$$\inf \tilde{a}_i = \left[\underset{i\in I}{\wedge} a_i^-, \underset{i\in I}{\wedge} a_i^+\right], \sup \tilde{a}_i = \left[\underset{i\in I}{\vee} a_i^-, \underset{i\in I}{\vee} a_i^+\right]$$

and put

$(1)\tilde{a}_1 \le \tilde{a}_2 \Leftrightarrow a_1^- \le a_2^-$ and $a_1^+ \le a_2^+,$

$(2)\tilde{a}_1 = \tilde{a}_2 \Leftrightarrow a_1^- = a_2^-$ and $a_1^+ = a_2^+,$

$(3)\tilde{a}_1 < \tilde{a}_2 \Leftrightarrow \tilde{a}_1 \le \tilde{a}_2$ and $\tilde{a}_1 \ne \tilde{a}_2,$

$(4)k\tilde{a} = \left[ka^-, ka^+\right],$ whenever $0 \le k \le 1.$

It is clear that $(D[0,1], \le, \vee, \wedge)$ is a complete lattice with $0 = [0,0]$ as least element and $1 = [1,1]$ as greatest element.

By an interval valued fuzzy set F on X we mean the set $F = \left\{\left(x, \left[\mu_F^-(x), \mu_F^+(x)\right]\right): x \in X\right\}.$ Where μ_F^- and μ_F^+ are fuzzy subsets of X such that $\mu_F^-(x) \le \mu_F^+(x)$ for all $x \in X.$ Put $\tilde{\mu}_F(x) = \left[\mu_F^-(x), \mu_F^+(x)\right].$ Then $F = \left\{\left(x, \tilde{\mu}_F(x)\right): x \in X\right\},$ where $\tilde{\mu}_F : X \to D[0,1].$

If A, B are two interval valued fuzzy subsets of X, then we define

$A \subseteq B$ if and only if for all $x \in X,$ $\mu_A^-(x) \le \mu_B^-(x)$ and $\mu_A^+(x) \le \mu_B^+(x),$

$A = B$ if and only if for all $x \in X,$ $\mu_A^-(x) = \mu_B^-(x)$ and $\mu_A^+(x) = \mu_B^+(x).$

Also, the union, intersection and complement are defined as follows: let A; B be two interval valued fuzzy subsets of X, then

$$A\cup B=\left\{\left(x,\left[\max\left\{\mu_A^-(x),\mu_B^-(x)\right\},\max\left\{\mu_A^+(x),\mu_B^+(x)\right\}\right]\right):x\in X\right\},$$

$$A\cap B=\left\{\left(x,\left[\min\left\{\mu_A^-(x),\mu_B^-(x)\right\},\min\left\{\mu_A^+(x),\mu_B^+(x)\right\}\right]\right):x\in X\right\},$$

$$A^c=\left\{\left(x,\left[\left\{1-\mu_A^-(x),1-\mu_A^+(x)\right\}\right]\right):x\in X\right\}.$$

According to Atanassov an interval valued intuitionistic fuzzy set on X is defined as an object of the form $A=\left\{\left(x,\tilde{\mu}_A(x),\tilde{\lambda}_A(x)\right):x\in X\right\}$, where $\tilde{\mu}_A(x)$ and $\tilde{\lambda}_A(x)$ are interval valued fuzzy sets on X such that $0\le\sup\tilde{\mu}_A(x)+\sup\tilde{\lambda}_A(x)\le 1$ for all $x\in X$.

For the sake of simplicity, in the following such interval valued intuitionistic fuzzy sets will be denoted by $A=\left(\tilde{\mu}_A,\tilde{\lambda}_A\right)$.

Interval Valued Intuitionistic (α,β) -fuzzy Hv-submodules

In this section we give the definition of interval valued intuitionistic (α,β)-fuzzy Hv-submodule and prove some related results.

Definition 3.1 An interval valued intuitionistic fuzzy set $A=\{\tilde{\mu}_A,\tilde{\lambda}_A\}$ in M is called an interval valued intuitionistic (α,β)-fuzzy H_v-submodule of M if for all $t,r\in(0,1]$,

$(1)\ \forall x,y\in M,\quad x_t\alpha\tilde{\mu}_A, y_r\alpha\tilde{\mu}_A\Rightarrow z_{t\wedge r}\beta\tilde{\mu}_A$ for all $z\in x+y$,

$(2)\ \forall x,a\in M,\quad x_t\alpha\tilde{\mu}_A, a_r\alpha\tilde{\mu}_A\Rightarrow (y\wedge z)_{t\wedge r}\beta\tilde{\mu}_A$ for some $y,z\in M$ with $x\in(a+y)\cap(z+a)$,

$(3)\ \forall x,y\in M,\quad y_t\alpha\tilde{\mu}_A\Rightarrow z_t\beta\tilde{\mu}_A$ for all $z\in x\cdot y$,

$(4)\ \forall x,y\in M,\quad x_t\bar{\alpha}\tilde{\lambda}_A, y_r\bar{\alpha}\tilde{\lambda}_A\Rightarrow z_{t\wedge r}\bar{\beta}\tilde{\lambda}_A$ for all $z\in x+y$,

$(5)\ \forall x,a\in M,\quad x_t\bar{\alpha}\tilde{\lambda}_A, a_r\bar{\alpha}\tilde{\lambda}_A\Rightarrow (y\wedge z)_{t\wedge r}\bar{\beta}\tilde{\lambda}_A$ for some $y,z\in M$ with $x\in(a+y)\cap(z+a)$,

$(6)\ \forall x,y\in M,\quad y_t\bar{\alpha}\tilde{\lambda}_A\Rightarrow z_{t\wedge r}\bar{\beta}\tilde{\lambda}_A$ for all $z\in x\cdot y$.

Lemma 3.2 Let $A=\{\tilde{\mu}_A,\tilde{\lambda}_A\}$ be an interval valued intuitionistic fuzzy set in M. Then for all $x\in M$ and $r\in(0,1]$, we have

$(1)\ x_t q\tilde{\mu}_A\Leftrightarrow x_t\,\bar{\in}\,\tilde{\mu}_A^c.$

$(2)\ x_t\in\vee q\tilde{\mu}_A\Leftrightarrow x_t\overline{\in\wedge q}\ \tilde{\mu}_A^c.$

Proof. (1) Let $x\in M$ and $r\in(0,1]$. Then, we have

$$\begin{aligned}x_t q\tilde{\mu}_A&\Leftrightarrow\tilde{\mu}_A(x)+t>1\\&\Leftrightarrow 1-\tilde{\mu}_A(x)<t\\&\Leftrightarrow\tilde{\mu}_A^c(x)<t\\&\Leftrightarrow x_t\,\bar{\in}\,\tilde{\mu}_A^c.\end{aligned}$$

(2) Let $x \in M$ and $r \in (0,1]$. Then, we have

$$\begin{aligned} x_t \in \vee q\tilde{\mu}_A &\Leftrightarrow x_t \in \tilde{\mu}_A \text{ or } x_t q\tilde{\mu}_A \\ &\Leftrightarrow \tilde{\mu}_A(x) \geq t \text{ or } \tilde{\mu}_A(x) + t > 1 \\ &\Leftrightarrow 1 - \tilde{\mu}_A^c(x) \geq t \text{ or } 1 - \tilde{\mu}_A^c(x) + t > 1 \\ &\Leftrightarrow x_t \overline{q}\tilde{\mu}_A^c \text{ or } x_t \,\overline{\in}\, \tilde{\mu}_A^c \\ &\Leftrightarrow x_t \overline{\in \wedge q}\tilde{\mu}_A^c. \end{aligned}$$

Theorem 3.3 If $A = \{\tilde{\mu}_A, \tilde{\lambda}_A\}$ is an interval valued intuitionistic $(\in, \in)$-fuzzy H_v-submodule of M then $A = \{\tilde{\mu}_A, \tilde{\lambda}_A\}$ is an interval valued intuitionistic fuzzy H_v-submodule of M.

Proof Condition (1). Let $x, y \in M$ and $\tilde{\mu}_A(x) \wedge \tilde{\mu}_A(y) = t$. Then $x_t, y_t \in \tilde{\mu}_A$. By condition (1) of definition 3.1, we have $z_t \in \tilde{\mu}_A$, $\forall z \in x + y$, and so $\tilde{\mu}_A(z) \geq t$, $\forall z \in x + y$. Consequently $\tilde{\mu}_A(x) \wedge \tilde{\mu}_A(y) = t \leq \bigwedge_{z \in x+y} \tilde{\mu}_A(z)$ for all $x, y \in M$.

Condition (2). Now let $x, a \in M$ and $\tilde{\mu}_A(x) \wedge \tilde{\mu}_A(a) = t$. Then $x_t, a_t \in \tilde{\mu}_A$. It follows from condition (2) of definition 3.1 that $(y \wedge z)_t \in \tilde{\mu}_A$, for some $y, z \in M$ with $x \in (a + y) \cap (z + a)$. Thus $y_t, z_t \in \tilde{\mu}_A$, for some $y, z \in M$ with $x \in (a + y) \cap (z + a)$. So, for all $x, a \in M$, there exist $y, z \in M$ such that $x \in (a + y) \cap (z + a)$ and $\tilde{\mu}_A(x) \wedge \tilde{\mu}_A(a) = t \leq \tilde{\mu}_A(y) \wedge \tilde{\mu}_A(z)$.

Condition (3). Let $x, y \in M$ and $\tilde{\mu}_A(y) = t$. Thus $y_t \in \tilde{\mu}_A$. From condition (3) of definition 3.1, we have $z_t \in \tilde{\mu}_A$ for all $z \in x \cdot y$, and so $\tilde{\mu}_A(z) \geq t$ for all $z \in x \cdot y$. This proves that $\tilde{\mu}_A(y) = t \leq \bigwedge_{z \in x \cdot y} \tilde{\mu}_A(z)$ for all $x, y \in M$.

Condition (4). Let $x, y \in M$ and $\tilde{\lambda}_A(x) \vee \tilde{\lambda}_A(y) = s$. If $s = 1$, then $\tilde{\lambda}_A(z) \leq 1 = s$ for all $z \in x + y$. It is easy to see that $\bigvee_{z \in x \cdot y} \tilde{\lambda}_A(z) \leq \tilde{\lambda}_A(x) \vee \tilde{\lambda}_A(y)$ for all $x, y \in M$. If $s < 1$ there exists a $t \in (0,1]$ such that $\tilde{\lambda}_A(x) \vee \tilde{\lambda}_A(y) = s < t$. Then $x_t, y_t \,\overline{\in}\, \tilde{\lambda}_A$. By condition (4) of definition 3.1, we have $z_t \,\overline{\in}\, \tilde{\lambda}_A$, $\forall z \in x + y$ and so $\tilde{\lambda}_A(z) < t$. Consequently $\bigvee_{z \in x \cdot y} \tilde{\lambda}_A(z) \leq \tilde{\lambda}_A(x) \vee \tilde{\lambda}_A(y)$ for all $x, y \in M$.

Condition (5). Now let $x, a \in M$ and $\tilde{\lambda}_A(x) \vee \tilde{\lambda}_A(a) = s$. If $s < 1$, there exists a $t \in (0,1]$ such that $\tilde{\lambda}_A(x) \vee \tilde{\lambda}_A(a) = s < t$. Then $x_t, a_t \,\overline{\in}\, \tilde{\lambda}_A$. By condition (5) of definition 3.1, we have $(y \wedge z)_t \,\overline{\in}\, \tilde{\lambda}_A$ for some $y, z \in M$ with $x \in (a + y) \cap (z + a)$. Hence $\tilde{\lambda}_A(y) < t$ and $\tilde{\lambda}_A(z) < t$. Thus $\tilde{\lambda}_A(y) \vee \tilde{\lambda}_A(z) < t$. This implies that for all $x, a \in M$, there exist $y, z \in M$ such that $x \in (a + y) \cap (z + a)$ and $\tilde{\lambda}_A(y) \leq \tilde{\lambda}_A(x) \vee \tilde{\lambda}_A(a)$. If $s = 1$ the proof is obvious.

Condition (6). Let $x, y \in M$ and $\tilde{\lambda}_A(y) = s$. If $s < 1$, there exists a $t \in (0,1]$ such that $\tilde{\lambda}_A(y) = s < t$. Thus $y_t \overline{\in} \tilde{\lambda}_A$. From condition (6) of definition 3.1, we have $z_t \overline{\in} \tilde{\lambda}_A$ for all $z \in x \cdot y$, and so $\tilde{\lambda}_A(z) < t$ for all $z \in x \cdot y$. Then $\tilde{\lambda}_A(z) < \tilde{\lambda}_A(y)$. This proves that $\underset{z \in x \cdot y}{\vee} \tilde{\lambda}_A(z) \le \tilde{\lambda}_A(y)$, for all $x, y \in R$. If $s = 1$ the proof is obvious.

Theorem 3.4 If $A = \{\tilde{\mu}_A, \tilde{\lambda}_A\}$ is an intuitionistic $(\in, \in \vee q)$ and $(\in, \in \wedge q)$-fuzzy H_v-submodule of M then $A = \{\tilde{\mu}_A, \tilde{\lambda}_A\}$ is an intuitionistic fuzzy H_v-submodule of M.
Proof The proof is similar to the proof of Theorem 3.3.

Theorem 3.5 If □$A = \left\{\tilde{\mu}_A, \tilde{\mu}_A^c\right\}$ is an interval valued intuitionistic (α, β)-fuzzy H_v-submodule of M if and only if □$A = \left\{\tilde{\mu}_A, \tilde{\mu}_A^c\right\}$ is an interval valued intuitionistic (α', β')-fuzzy H_v-submodule of M, where $\alpha \in \{\in, q\}$ and $\beta \in \{\in, q, \in \vee q, \in \wedge q\}$.

Proof We only prove the case of $(\alpha, \beta) = (\in, \in \vee q)$. The others are analogous. Let □$A = \left\{\tilde{\mu}_A, \tilde{\mu}_A^c\right\}$ be an intuitionistic $(\in, \in \vee q)$-fuzzy H_v-submodule of M.

Condition (1). Let $x, y \in M$ and $t, r \in (0,1]$ be such that $x_t, y_r q \tilde{\mu}_A$. It follows from Lemma 3.2 that $x_t, y_r \overline{\in} \tilde{\mu}_A^c$. Since $\tilde{\mu}_A^c$ is an anti $(\in, \in \vee q)$-fuzzy H_v-submodule of M. Thus by condition (4) of definition 3.1, we have $z_{t \wedge r} \overline{\in \vee q} \tilde{\mu}_A^c$ for all $z \in x + y$. By Lemma 3.2, this is equivalence with $z_{t \wedge r} \in \wedge q \tilde{\mu}_A$ for all $z \in x + y$. Thus condition of (1) of definition 3.1 is valid.

Condition (2). Suppose that $x, a \in M$ and $t, r \in (0,1]$ be such that $x_t, a_r q \tilde{\mu}_A$. By Lemma 3.2, we have $x_t, a_r q \tilde{\mu}_A$ iff $x_t, a_r \overline{\in} \tilde{\mu}_A^c$. By hypotheses, $\tilde{\mu}_A^c$ is an anti $(\in, \in \vee q)$-fuzzy H_v-submodule of M. Thus by condition (5) of definition 3.1, we have $(y \wedge z)_{t \wedge r} \overline{\in \vee q} \tilde{\mu}_A^c$, for some $y, z \in M$ with $x \in (a + y) \cap (z + a)$. This is equivalence with $y_{t \wedge r} \overline{\in \vee q} \tilde{\mu}_A^c$ and $z_{t \wedge r} \overline{\in \vee q} \tilde{\mu}_A^c$, for some $y, z \in M$ with $x \in (a + y) \cap (z + a)$. By Lemma 3.2, it is easy to see that $y_{t \wedge r} \in \wedge q \tilde{\mu}_A$ and $z_{t \wedge r} \in \wedge q \tilde{\mu}_A$, for some $y, z \in M$ with $x \in (a + y) \cap (z + a)$ if and only if $(y \wedge z)_{t \wedge r} \in \wedge q \tilde{\mu}_A$, for some $y, z \in M$ with $x \in (a + y) \cap (z + a)$. Thus condition of (2) of definition 3.1 is valid.

Condition (3). Let $x, y \in M$ and $t \in (0,1]$ be such that $y_t q \tilde{\mu}_A$. It follows from Lemma 3.2 that $y_t \overline{\in} \tilde{\mu}_A^c$. Since □$A = \left\{\tilde{\mu}_A, \tilde{\mu}_A^c\right\}$ is an intuitionistic $(\in, \in \vee q)$-fuzzy H_v-submodule of M. Thus by condition (6) of definition 3.1, we have $z_t \overline{\in \vee q} \tilde{\mu}_A^c$ for all $z \in x \cdot y$. It is equivalence with $z_t \in \wedge q \tilde{\mu}_A$ for all $z \in x \cdot y$. Which verify conditions (3) of definition 3.1.

Condition (4). Let $x, y \in M$ and $t, r \in (0,1]$ be such that $x_t, y_r \overline{q} \tilde{\mu}_A^c$. It follows from Lemma 3.2 that $x_t, y_r \overline{q} \tilde{\mu}_A^c$ iff $x_t, y_r \in \tilde{\mu}_A$. Since □$A = \left\{\tilde{\mu}_A, \tilde{\mu}_A^c\right\}$ is an intuitionistic $(\in, \in \vee q)$-fuzzy H_v-submodule of M. Thus by condition (1) of definition 3.1, we have $z_{t\wedge r} \in \vee q\tilde{\mu}_A$ for all $z \in x+y$. By Lemma 3.2, this is equivalence with $z_{t\wedge r}\overline{\in \wedge q}\tilde{\mu}_A^c$ for all $z \in x+y$. Thus condition of (4) of definition 3.1 is valid.

Condition (5). Suppose that $x, a \in M$ and $t, r \in (0,1]$ be such that $x_t, a_r \overline{q}\tilde{\mu}_A^c$. This is equivalence with $x_t, a_r \in \tilde{\mu}_A$. By hypotheses, $\tilde{\mu}_A$ is an $(\in, \in\vee q)$-fuzzy H_v-submodule of M. Thus by condition (2) of definition 3.1, we have $(y \wedge z)_{t\wedge r} \in \vee q\tilde{\mu}_A$, for some $y, z \in M$ with $x \in (a+y)\cap(z+a)$, and so $y_{t\wedge r} \in \vee q\tilde{\mu}_A$, and $z_{t\wedge r} \in \vee q\tilde{\mu}_A$, for some $y, z \in M$ with $x \in (a+y)\cap(z+a)$. It follows from Lemma 3.2 that $y_{t\wedge r}\overline{\in \wedge q}\tilde{\mu}_A^c$ and $z_{t\wedge r}\overline{\in \wedge q}\tilde{\mu}_A^c$ for some $y, z \in M$ with $x \in (a+y)\cap(z+a)$ if and only if $(y \wedge z)_{t\wedge r}\overline{\in \wedge q}\tilde{\mu}_A^c$, for some $y, z \in M$ with $x \in (a+y)\cap(z+a)$. Thus condition of (5) of definition 3.2 is valid.

Condition (6). Let $x, y \in M$ and $t \in (0,1]$ be such that $y_t \overline{q}\tilde{\mu}_A^c$. Then, we have $y_t \in \tilde{\mu}_A$. Since $\square A = \{\tilde{\mu}_A, \tilde{\mu}_A^c\}$ is an interval valued intuitionistic $(\in, \in\vee q)$-fuzzy H_v-submodule of M. Thus by condition (3) of definition 3.1, we have $z_t \in \vee q\tilde{\mu}_A$ for all $z \in x \cdot y$. It is equivalence with $z_t\overline{\in \wedge q}\tilde{\mu}_A^c$ for all $z \in x \cdot y$. Which verify conditions (6) of definition 3.1.

Theorem 3.6 If $\Diamond A = \{\tilde{\lambda}_A^c, \tilde{\lambda}_A\}$ is an interval valued intuitionistic (α, β)-fuzzy H_v-submodule of M if and only if $\Diamond A = \{\tilde{\lambda}_A^c, \tilde{\lambda}_A\}$ is an interval valued intuitionistic (α', β')-fuzzy H_v-submodule of M, where $\alpha \in \{\in, q\}$ and $\beta \in \{\in, q, \in\vee q, \in\wedge q\}$.

Proof The proof is similar to the proof of Theorem 3.5.

Theorem 3.7 If $A = \{\tilde{\mu}_A, \tilde{\lambda}_A\}$ is an interval valued intuitionistic (α, β)-fuzzy H_v-submodule of M if and only if $\tilde{\mu}_A$ is an (α, β)-fuzzy H_v-submodule of M and $\tilde{\lambda}_A^c$ is an (α', β')-fuzzy H_v-submodule of M, where $\alpha \in \{\in, q\}$ and $\beta \in \{\in, q, \in\vee q, \in\wedge q\}$.

Proof We only prove the case of $(\alpha, \beta) = (\in, \in\vee q)$. The others are analogous. It is sufficient to show that, $\tilde{\lambda}_A^c$ is an $(q, \in\wedge q)$-fuzzy H_v-submodule of M if and only if $\tilde{\lambda}_A$ is an anti $(\in, \in\vee q)$-fuzzy H_v-submodule of M. This is true, because $x_t q\tilde{\lambda}_A \Leftrightarrow x_t \overline{\in} \tilde{\lambda}_A^c$ and $x_t \in \wedge q\tilde{\lambda}_A \Leftrightarrow x_t\overline{\in \vee q}\,\tilde{\lambda}_A^c$, $\forall x \in M$ and $t \in (0,1]$.

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